OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4723**

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 2 3 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1

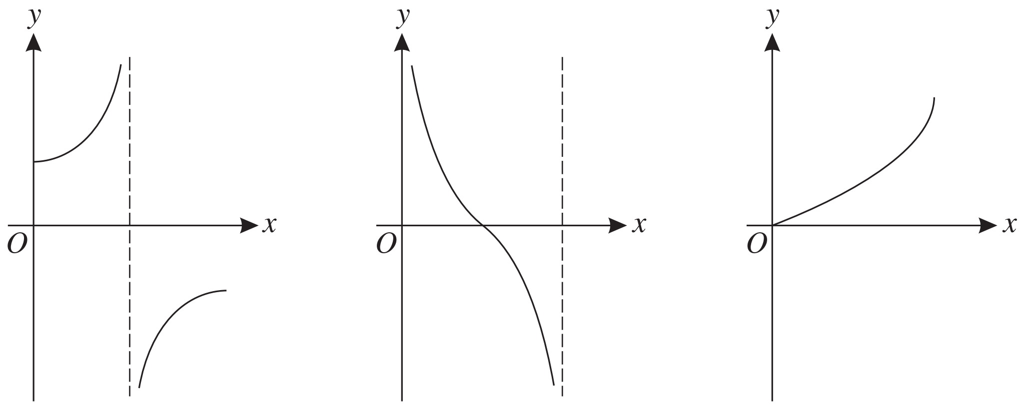


Fig. 1 **Fig. 2** **Fig. 3**

Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

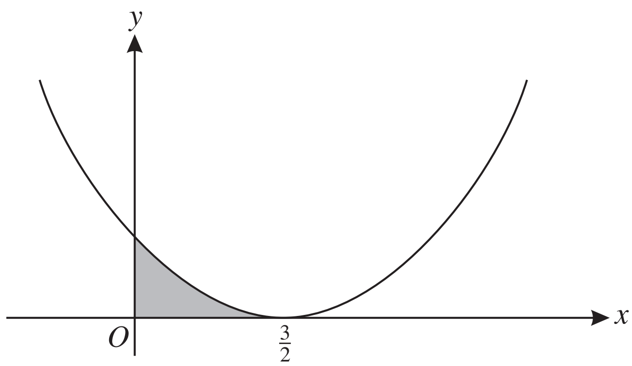
State which equation corresponds to

(i) Fig. 1, **[1]**

(ii) Fig. 2, **[1]**

(iii) Fig. 3. **[1]**

2



The diagram shows the curve with equation $y = (2x - 3)^2$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = 0$. Find the exact volume obtained when the shaded region is rotated completely about the x-axis. **[5]**

3 The angles α and β are such that

$$\tan\alpha = m + 2 \quad \text{and} \quad \tan\beta = m,$$

where m is a constant.

(i) Given that $\sec^2\alpha - \sec^2\beta = 16$, find the value of m. **[3]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

4 It is given that $\int_a^{3a} (e^{3x} + e^x)\,dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$. **[5]**

(ii) Use an iterative process, based on the equation in part **(i)**, to find the value of a correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process. **[4]**

5 The functions f and g are defined for all real values of x by

$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = 3x + 7.$$

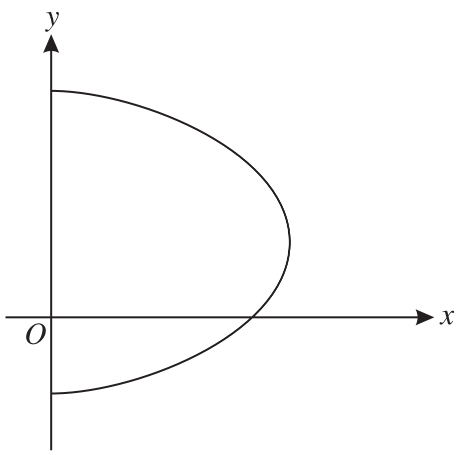
Find the exact coordinates of the point at which

(i) the graph of $y = fg(x)$ meets the x-axis, **[3]**

(ii) the graph of $y = g(x)$ meets the graph of $y = g^{-1}(x)$, **[3]**

(iii) the graph of $y = |f(x)|$ meets the graph of $y = |g(x)|$. **[4]**

6



The diagram shows the curve with equation $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. **[2]**

(ii) Hence find the equation of the tangent to the curve at the point (7, 3), giving your answer in the form $y = mx + c$. **[5]**

7 **(i)** Express $8\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

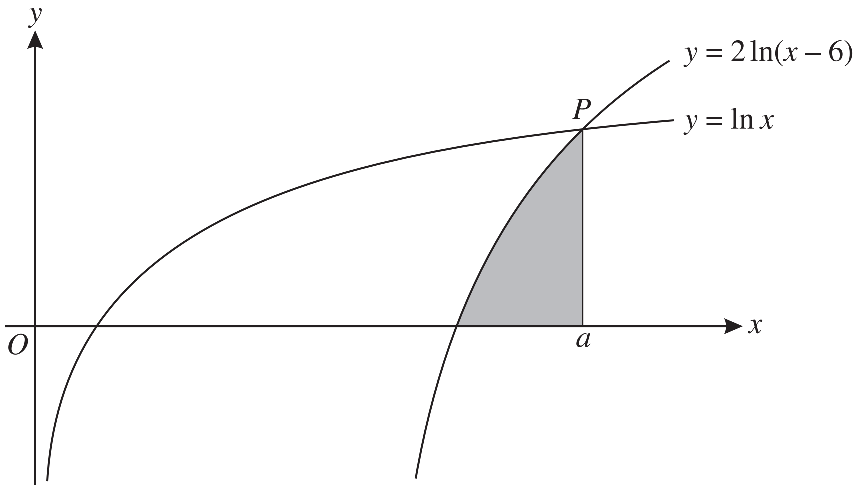
(a) solve, for $0° < \theta < 360°$, the equation $8\sin\theta - 6\cos\theta = 9$, **[4]**

(b) find the greatest possible value of

$$32\sin x - 24\cos x - (16\sin y - 12\cos y)$$

as the angles x and y vary. **[3]**

8



The diagram shows the curves $y = \ln x$ and $y = 2\ln(x - 6)$. The curves meet at the point P which has x-coordinate a. The shaded region is bounded by the curve $y = 2\ln(x - 6)$ and the lines $x = a$ and $y = 0$.

(i) Give details of the pair of transformations which transforms the curve $y = \ln x$ to the curve $y = 2\ln(x - 6)$. **[3]**

(ii) Solve an equation to find the value of a. **[4]**

(iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region. **[3]**

9 **(a)** Show that, for all non-zero values of the constant k, the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point. **[5]**

(b) Show that, for all non-zero values of the constant m, the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points. **[7]**

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RECOGNISING ACHIEVEMENT

4723 Core Mathematics 3

1 (i)	State $y = \sec x$	B1		
(ii)	State $y = \cot x$	B1		
(iii)	State $y = \sin^{-1} x$	B1	3	
			$\boxed{3}$	
2	Either: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Obtain integral of form $k(2x-3)^5$	M1		any constant k involving π or not
	Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$	A1		
	Attempt evaluation using 0 and $\frac{3}{2}$	M1		subtraction correct way round
	Obtain $\frac{243}{10}\pi$	A1	5	or exact equiv
	Or: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Expand and obtain integral of order 5	M1		with at least three terms correct
	Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$	A1		with or without π
	Attempt evaluation using (0 and) $\frac{3}{2}$	M1		
	Obtain $\frac{243}{10}\pi$	A1	(5)	or exact equiv
			$\boxed{5}$	
3 (i)	Attempt use of identity for $\sec^2\alpha$	M1		using $\pm\tan^2\alpha \pm 1$
	Obtain $1+(m+2)^2-(1+m^2)$	A1		absent brackets implied by subsequent correct working
	Obtain $4m + 4 = 16$ and hence $m = 3$	A1	3	
(ii)	Attempt subn in identity for $\tan(\alpha+\beta)$	M1		using $\dfrac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$
	Obtain $\dfrac{5+3}{1-15}$ or $\dfrac{m+2+m}{1-m(m+2)}$	A1√		following their m
	Obtain $-\frac{4}{7}$	A1	3	or exact equiv
			$\boxed{6}$	
4 (i)	Obtain $\frac{1}{3}e^{3x} + e^x$	B1		
	Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$	B1		or equiv
	Equate definite integral to 100 and attempt rearrangement	M1		as far as $e^{9a} = \ldots$
	Introduce natural logarithm	M1		using correct process
	Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$	A1	5	AG; necessary detail needed
(ii)	Obtain correct first iterate	B1		allow for 4 dp rounded or truncated
	Show correct iteration process	M1		with at least one more step
	Obtain at least three correct iterates in all	A1		allowing recovery after error
	Obtain 0.6309	A1	4	following at least three correct steps; answer required to exactly 4 dp
	[$0.6 \to 0.631269 \to 0.630884 \to 0.630889$]			
			$\boxed{9}$	

5 (i)	Either: Show correct process for comp'n	M1		correct way round and in terms of x
	Obtain $y = 3(3x+7) - 2$	A1		or equiv
	Obtain $x = -\frac{19}{9}$	A1	3	or exact equiv; condone absence of $y = 0$
	Or: Use fg(x) = 0 to obtain $g(x) = \frac{2}{3}$	B1		
	Attempt solution of $g(x) = \frac{2}{3}$	M1		
	Obtain $x = -\frac{19}{9}$	A1	(3)	or exact equiv; condone absence of $y = 0$
(ii)	Attempt formation of one of the equations $3x+7 = \frac{x-7}{3}$ or $3x+7 = x$ or $\frac{x-7}{3} = x$	M1		or equiv
	Obtain $x = -\frac{7}{2}$	A1		or equiv
	Obtain $y = -\frac{7}{2}$	A1√	3	or equiv; following their value of x
(iii)	Attempt solution of modulus equation	M1		squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
	Obtain $-12x + 4 = 42x + 49$ or $3x - 2 = -3x - 7$	A1		or equiv
	Obtain $x = -\frac{5}{6}$	A1		or exact equiv; as final answer
	Obtain $y = \frac{9}{2}$	A1	4	or equiv; and no other pair of answers
			10	

6 (i)	Obtain derivative $k(37+10y-2y^2)^{-\frac{1}{2}}f(y)$	M1		any constant k; any linear function for f
	Obtain $\frac{1}{2}(10-4y)(37+10y-2y^2)^{-\frac{1}{2}}$	A1	2	or equiv
(ii)	Either: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	*M1		
	Take reciprocal of expression/value	*M1		and without change of sign
	Obtain -7 for gradient of tangent	A1		
	Attempt equation of tangent	M1		dep *M *M
	Obtain $y = -7x + 52$	A1	5	and no second equation
	Or: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	M1		
	Attempt formation of eq'n $x = m'y + c$	M1		where m' is attempt at $\frac{dx}{dy}$
	Obtain $x - 7 = -\frac{1}{7}(y-3)$	A1		or equiv
	Attempt rearrangement to required form	M1		
	Obtain $y = -7x + 52$	A1	(5)	and no second equation
			7	

7 (i)	State $R = 10$	B1	or equiv
	Attempt to find value of α	M1	implied by correct answer or its complement; allow sin/cos muddles
	Obtain 36.9 or $\tan^{-1}\frac{3}{4}$	A1 3	or greater accuracy 36.8699…
(ii)(a)	Show correct process for finding one angle	M1	
	Obtain (64.16 + 36.87 and hence) 101	A1	or greater accuracy 101.027…
	Show correct process for finding second angle	M1	
	Obtain (115.84 + 36.87 and hence) 153	A1√ 4	following their value of α; or greater accuracy 152.711…; and no other between 0 and 360
(b)	Recognise link with part **(i)**	M1	signalled by 40 … – 20 …
	Use fact that maximum and minimum values of sine are 1 and –1	M1	may be implied; or equiv
	Obtain 60	A1 3	
		10	
8 (i)	Refer to translation and stretch	M1	in either order; allow here equiv informal terms such as 'move', …
	State translation in x direction by 6	A1	or equiv; now with correct terminology
	State stretch in y direction by 2	A1 3	or equiv; now with correct terminology
	[SC: if M0 but one transformation completely correct, give B1]		
(ii)	State $2\ln(x-6)=\ln x$	B1	or $2\ln(a-6)=\ln a$ or equiv
	Show correct use of logarithm property	*M1	
	Attempt solution of 3-term quadratic	M1	dep *M
	Obtain 9 only	A1 4	following correct solution of equation
(iii)	Attempt evaluation of form $k(y_0+4y_1+y_2)$	M1	any constant k; maybe with $y_0=0$ implied
	Obtain $\frac{1}{3}\times 1(2\ln 1+8\ln 2+2\ln 3)$	A1	or equiv
	Obtain 2.58	A1 3	or greater accuracy 2.5808…
		10	
9 (a)	Attempt use of quotient rule	*M1	or equiv; allow numerator wrong way round and denominator errors
	Obtain $\dfrac{(kx^2+1)2kx-(kx^2-1)2kx}{(kx^2+1)^2}$	A1	or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1	
	Equate numerator of first derivative to zero	M1	dep *M
	State $x=0$ <u>or</u> refer to $4kx$ being linear <u>or</u> observe that, with $k\neq 0$, only one sol'n	A1√ 5	AG or equiv; following numerator of form $k'kx=0$, any constant k'

(b)	Attempt use of product rule	*M1		
	Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$	A1		or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1		dep *M
	Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that e^{mx} cannot be zero	A1		
	Attempt use of discriminant	M1		using correct b^2-4ac with their a, b, c
	Simplify to obtain m^4+4	A1		or equiv
	Observe that this is positive for all m and hence two roots	A1	7	or equiv; AG
			12	